Squeezed light generation for gravitational wave detection

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The generation of squeezed light has emerged as a critical technology for enhancing the sensitivity of gravitational wave detectors such as LIGO. Squeezed light, achieved through processes like parametric down-conversion in optical parametric oscillators, reduces quantum noise by manipulating the uncertainties in the conjugate operators of the photon wavefunction. This review explores the theoretical foundations of squeezed light, its practical generation techniques, and its application in LIGO to minimize shot noise and radiation pressure noise. The deployment of squeezed light has significantly improved LIGO's detection capabilities, enabling the observation of faint astrophysical events. As advancements continue, the optimization of squeezed light sources and configurations promises even greater sensitivity, expanding our understanding of the cosmos.

I. SQUEEZED LIGHT

Light in a harmonic oscillator potential, i.e. in a laser, typically exists in a coherent state, in which the position and momentum uncertainties are equal.

$$\psi_0(x) = \frac{1}{\pi^{1/4}} e^{-x^2/2} \tag{1}$$

Simply by observing that "squeezing" the wavefunction in position space leads to a "spreading" in momentum space, while adhering to the general Heisenberg uncertainty principle, we note the uncertainty of each operator: [1]

$$\begin{aligned}\psi_R(x) &= \frac{\sqrt{R}}{\pi^{1/4}} e^{-(Rx)^2/2} \\ \tilde{\psi}_R(p) &= \frac{1}{\pi^{1/4}\sqrt{R}} e^{-(p/R)^2/2}\end{aligned} \tag{2}$$

$$\begin{aligned}\langle \Delta x^2 \rangle &= \frac{1}{2R^2} \\ \langle \Delta p^2 \rangle &= \frac{R^2}{2}\end{aligned} \tag{3}$$

So we can achieve a decrease in uncertainty in one operator through an increase in uncertainty in its conjugate operator.

More formally, we may define a "squeezing operator" as follows:

$$\hat{S}(\zeta) = e^{(\zeta \hat{a}^2 - \zeta^* \hat{a}^{\dagger 2})/2} \tag{4}$$

with its action on the raising and lowering operators given by:

$$\begin{aligned}\hat{a}(t) &= \hat{a}(0)\cosh r - \hat{a}^\dagger(0)\sinh r \\ \hat{a}^\dagger(t) &= \hat{a}^\dagger(0)\cosh r - \hat{a}(0)\sinh r\end{aligned} \tag{5}$$

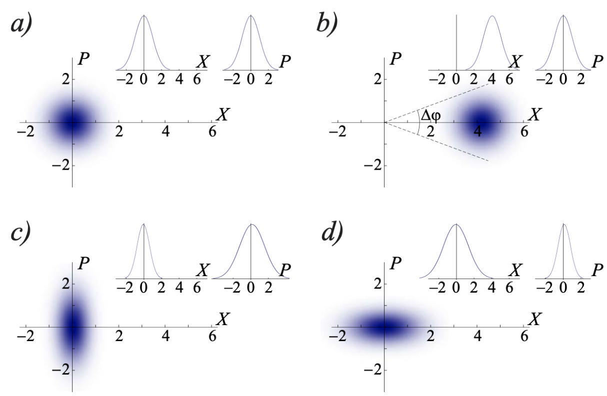

FIG. 1: Probability density diagrams in position-momentum phase space of 1) the ground state, 2) a coherent state (ground state translated in x), 3) a position-squeezed state, and 4) a momentum-squeezed state [1]

I.1. Generation through parametric down-conversion

Squeezed states are generated in LIGO through parametric down-conversion taking place within a bowtie optical parametric oscillator (OPO) to amplify the interaction.

I.1.1. Parametric down-conversion

In parametric down-conversion, a high-energy photon - the pump photon in the cavity - travels through a non-linear medium with second-order susceptibility $\chi^{(2)}$ and splits into two lower-energy photons, the "signal" and "idler" photons, in a squeezed state. [2]

For the conversion to be successful, both energy and momentum should be conserved:

$$\begin{aligned}\omega_p &= \omega_i + \omega_s \\ \mathbf{k_p} &= \mathbf{k_i} + \mathbf{k_s}\end{aligned} \tag{6}$$

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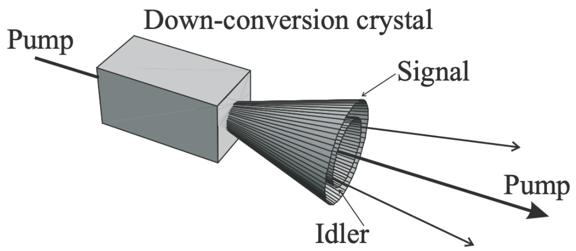

FIG. 2: Schematic of parametric down-conversion, which in the degenerate case produces an output signal and idler photon each at half the frequency of the pump photon. [3]

For simplicity we may assume the output photons are degenerate, i.e. indistinguishable in all observables, so that $\hat{a}_s = \hat{a}_i = \hat{a}$; qualitatively the same squeezing result arises in the nondegenerate case, but gravitational wave detectors employ the degenerate process anyway.[1] By selecting for the resulting signal and idler photons at angles which satisfy this relation - in LIGO, type I down-conversion is used, which selects for signal/idler photons in phase with each other but orthogonal to the pump field - the Hamiltonian becomes:[3]

$$\hat{H} = \chi^{(2)}\hat{a}_p(\hat{a}^\dagger)^2 + c.c. = \hbar\chi^{(2)}E_p(\hat{a}^\dagger)^2 + c.c. \quad (7)$$

The unitary time evolution operator corresponding to this Hamiltonian is just the squeezing operator in equation 4. If the initial state is the vacuum, or more precisely $|1\rangle_p \otimes |0\rangle_s \otimes |0\rangle_i$, then an evolution in time of the position and momentum operators applied to the initial state $|0\rangle$ of the output photon gives:[1]

$$\begin{aligned} \hat{x}(t) &= \hat{S}^\dagger(r)\hat{x}(0)\hat{S}(r) = \hat{x}(0)e^{-r} \\ \hat{p}(t) &= \hat{S}^\dagger(r)\hat{p}(0)\hat{S}(r) = \hat{p}(0)e^{r} \end{aligned} \quad (8)$$

So both the resulting signal and idler photons are in a position-squeezed state. Because this conversion process is very inefficient, a strong pump field is needed. Increasing the time t that the state is acted upon by the nonlinear medium also helps to increase the squeeze factor and leads to a further reduction in noise. Placing the nonlinear medium within a resonant cavity accomplishes both these things. [4, 5]

I.1.2. Optical parametric oscillator

The optical parametric oscillator is precisely this nonlinear medium situated between two mirrors, each with a coating to preferentially transmit the pump field at ω_p. In this way the strength of the pump field in the cavity is kept approximately constant.[6] The pump field both drives the generation of the squeezed vacuum, and provides frequency feedback for control over the OPO cavity lengths.[7] The OPO is resonant with both the pump field - at the higher frequency ω_p - and the squeezed field, at $\omega_s = \omega_p/2$ (in the degenerate case).[7] Thus, the input pump field and resultant squeezed field are both maintained while other frequencies are damped.

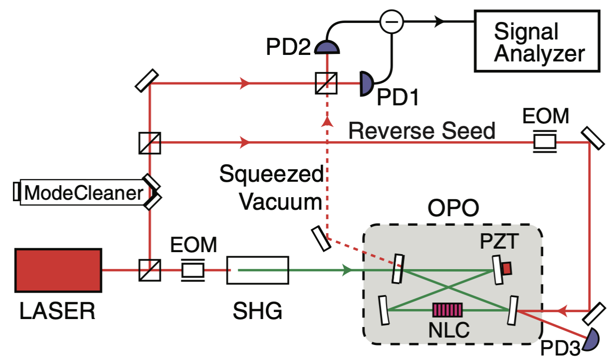

FIG. 3: Schematic of overall configuration, including optical parametric oscillator in a bowtie configuration, as exists in LIGO. [8]

II. LIGO EXPERIMENTAL SETUP

The Laser Interferometer Gravitational-Wave Observatory (LIGO) has detected over 90 gravitational wave events by detecting miniscule changes in length of its interferometer arms. Because the noise in the interferometer output signal depends on the vacuum state in the cavity, injecting a squeezed vacuum improves overall sensitivity; this method was first used for the above-700Hz range and later applied as low as 150Hz, the audiofrequency region where the gravitational wave signatures of several astrophysical objects such as black holes lie.[9]

In the operation of LIGO, an input laser beam travels through a beam splitter and travels through the two arms of the interferometer. Incidence of a gravitational wave upon the interferometer manifests as a distortion of spacetime that causes one arm to contract in length and the other to expand. This slight change in length also produces a relative phase shift between the modes of light in the two arms, which appears in the interferometer output as the presence of a signal, whereas in the absence of a gravitational wave the two modes would perfectly destructively interfere and result in no detected light. Also, to isolate the effect of a gravitational wave from background vibrations, e.g. seismic waves or passing cars, the mirrors of the interferometer are massive pendula.

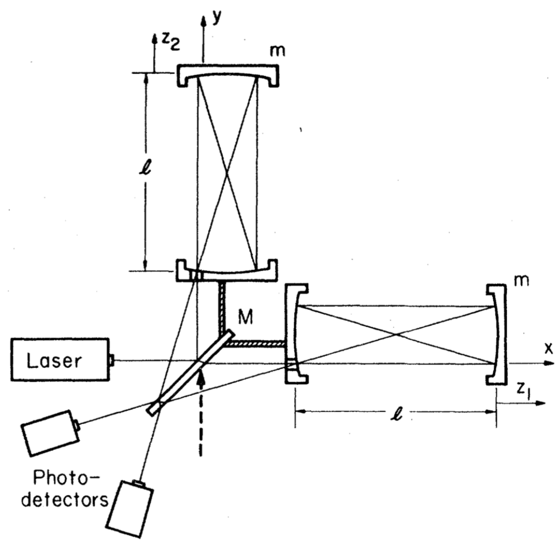

FIG. 4: A simple schematic of a Michelson interferometer as used in LIGO. [5]

II.1. Interferometry

As seen in the simplified schematic in figure 4, the basic principle of the interferometer relies on an input laser beam being split through a beam splitter and each split beam then bouncing many times between the mirrors of the interferometer arm, amplifying any difference in distance travelled between the beams.

Note that the other port into the beam splitter is empty; the vacuum enters here. Fluctuations in the vacuum and laser modes do not by themselves contribute to the final error since these fluctuations are split evenly between the two arms of the interferometer and can be accounted for by measuring at a particular location in the interference pattern. However, the interference between the laser and vacuum fluctuations does contribute unavoidably to noise.[5]

Thus, squeezing the vacuum is desirable to reduce this noise, which results directly from quantum fluctuations in the vacuum modes of the electromagnetic field. The mathematical reasoning for how this occurs and the different types of noise present are described in section III.

III. QUANTIFYING QUANTUM NOISE

There are two main quantum sources of noise in the LIGO output, stemming from the position or momentum uncertainty of the photon wavefunction, respectively. In the following calculations, we distinguish $\hat{a}_p$ as the incident light from the pump field/driving laser field from $\hat{b}_p$ the outgoing light along $-x$ (after reflecting in the interferometer arm n times). Similarly, $\hat{a}_s$ is the incident vacuum light, corresponding to a signal photon from the parametric down-conversion process, while $\hat{b}_s$ is the outgoing light along $-y$.

By considering the electric field in each of these modes, we can relate them as follows, with Δ the total phase shift and $\pi/2$ the phase shift in the beam splitter: [5]

$$\begin{aligned} E_p^{out} &= e^{-i\Delta}\frac{E_p^{in} - iE_s^{in}}{\sqrt{2}} \\ E_s^{out} &= e^{-i\Delta}\frac{E_s^{in} - iE_p^{in}}{\sqrt{2}} \end{aligned} \tag{9}$$

III.1. Quantum radiation pressure noise

Radiation pressure noise refers to the effect caused by recoil of mirrors due to the radiation pressure from quantum fluctuations in photon flux, and is caused by the amplitude quadrature of vacuum EM fluctuations.[9]

The difference between momentum transferred to each mirror is proportional to the difference between number of photons in the vacuum mode and number of photons in the pump mode:

$$\begin{aligned} \hat{P} &= \frac{2n\hbar\omega_s}{c}(\hat{b}_s^\dagger\hat{b}_s - \hat{b}_p^\dagger\hat{b}_p) \\ &= -i\frac{2n\hbar\omega_s}{c}(\hat{a}_s^\dagger\hat{a}_p - \hat{a}_p^\dagger\hat{a}_s) \end{aligned} \tag{10}$$

Written this way, it is clear that the source of this noise depends only on the interference between the s and p modes. Evaluated on a vacuum state squeezed by $\zeta = re^{i\phi}$, the variance in P is:[5]

$$(\Delta P)^2 = (\frac{2n\hbar\omega_s}{c})^2(\alpha^2 e^{2r} + \sinh^2(r)) \tag{11}$$

This grows for $r > 0$ (more phase squeezing) and shrinks for $r < 0$ (less phase squeezing).

III.2. Shot noise

Shot noise arises from statistical fluctuations in the arrival time of photons at the interferometer output. This is caused by the phase quadrature of vacuum EM fluctuations.[9] As the laser power increases, then, the shot noise decreases, while the radiation pressure noise increases.

Following a similar analysis as above, we may find an error in z (the deduced interferometer arm length difference):[5]

$$(\Delta z)^2 = (\frac{c}{2n\omega_s})^2(\frac{e^{-2r}}{\alpha^2} + \frac{\sinh^2 r}{\alpha^4}) \tag{12}$$

Now, for greater squeezing (r¿0), this error is reduced while for less squeezing (r¡0) this error is amplified.

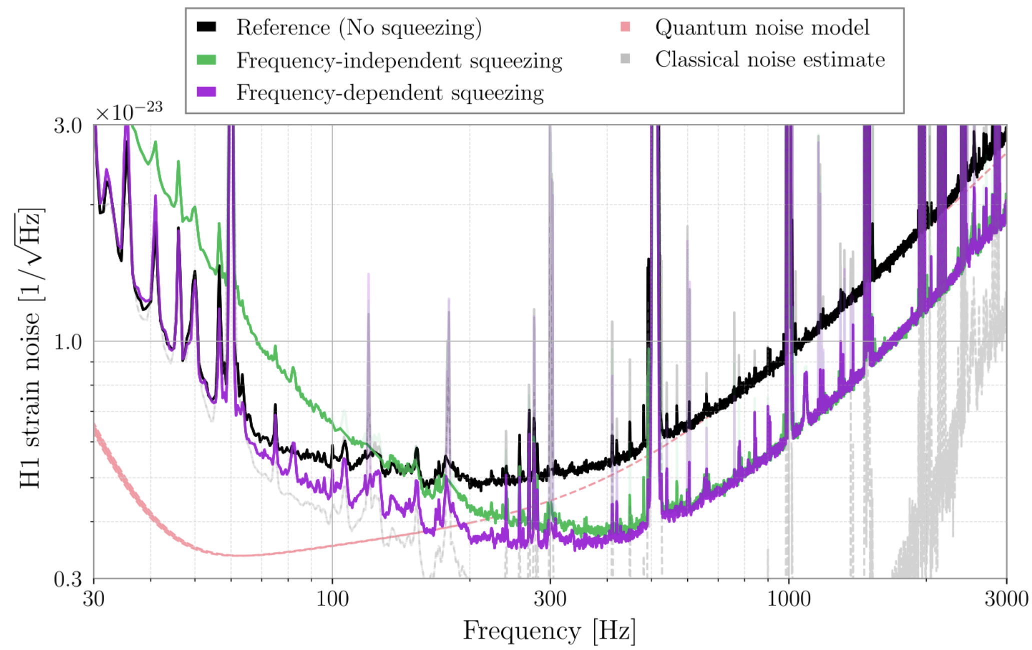

FIG. 5: In LIGO's Hanford interferometer, using frequency-dependent squeezing, noise is reduced at lower frequencies where radiation pressure noise was previously dominant, without sacrificing fidelity at higher frequencies where shot noise is dominant.[10]

III.3. Frequency-dependent squeezing in LIGO

The analysis above reveals that there is an inherent tradeoff in squeezing the vacuum state between decreasing photon counting noise and decreasing radiation pressure noise. For a time LIGO simply optimized the input laser power to achieve the minimum possible combined shot noise and radiation pressure noise, so their sensitivity was limited by the fact that a more powerful laser, while decreasing shot noise, would have made radiation pressure noise a larger factor.[5]

However, shot noise and radiation pressure noise are important at different frequencies, so this problem can also be addressed through frequency-dependent squeezing. In theory, this occurs by preparing vacuum states squeezed in amplitude at frequencies where radiation pressure noise dominates, and squeezed in phase at frequencies where shot noise dominates. Experimentally, this is achieved by generating a uniformly squeezed vacuum state which incurs a phase shift through reflection in a filter cavity.[10]

IV. OUTLOOK

The generation of squeezed light for use in gravitational wave detection experiments has advanced significantly in recent years. As more GW detection experiments of higher sensitivity are planned, the science behind squeezed light generation advances to allow detection of farther and weaker signals. In the future, beyond reducing optical losses or transmission losses in the equipment, different lasers and cavity designs are also being developed for squeezing light at the frequencies needed to probe new parts of the GW spectrum.[11]

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